Analytical expressions of the front shape of non-quasi-neutral plasma expansions with anisotropic electron pressures

Yongsheng Huang,[1,2,*] Yijin Shi,[1] Yuanjie Bi,[1,2] Xiaojiao Duan,[1] Naiyan Wang,[1] Xiuzhang Tang,[1] and Zhe Gao[2]
[1]*China Institute of Atomic Energy, Beijing 102413, China*
[2]*Department of Engineering Physics, Tsinghua University, Beijing 100084, China*
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An analytical expression is proposed to describe the front shape of a non-quasi-neutral plasma expansion with anisotropic electron pressures. It is of significance in the study of ultrashort plasma expansions generated from laser-foil interactions and anisotropic astroplasma expansions in space science. It is found that the plasma front shape depends on the relationship between the ratio of the longitudinal and the transverse temperature of hot electrons κ^2 and the electron-ion mass ratio μ. For $\kappa^2 \in (\mu, 1]$, the ion front is a part of an ellipse and the major axis is in the lower-temperature axis. For $\kappa^2 \leq \mu$, the ion front is composed by a part of a hyperbolic and a small pointed projection at the center. In the strongly anisotropic region, there is an ultrashort anomalous plasma emission of tens of femtoseconds at the angle of near 90°. The ion-velocity distribution and angular-energy distribution at the ion front have also been given. Particularly, anomalous positron emissions exist in the electron-positron plasma anisotropic expansion.

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I. INTRODUCTION

The ultrafast energetic plasma emission from laser-matter interactions offers new possibilities for compact accelerators [1], fast ignition for inertial-confinement fusion [2] due to great advances of laser technology. In the first time interval of several picoseconds, the plasma generated from laser-material interactions can achieve local thermodynamic equilibrium and hydrodynamical equations are adequate. However, the plasma has no time to reach macrothermal equilibrium. Electrons generated from the laser-solid interactions are anisotropic, generally, the plasma is non-quasi-neutral and the charge-separation field is strong in the ultrashort-time interval. Therefore, the two three-dimensional analytic kinetic theories under the quasineutral condition [3,4] are not suitable in the ultrashort time interval. Neglecting the strong charge-separation field, the two-dimension anisotropic plasma expansion models [5,6] are also inadequate. Although we proposed a two-dimension self-similar solution for describing non-quasi-neutral plasma expansions [7], the electron pressure was assumed isotropic and also improper in the time interval. In the ultrashort time interval, the field and the pressure of plasmas codetermine the plasma expansion process.

Almost no collisions are in the interval due to the high electron temperature larger than tens of keV; therefore, the energy transport between electron and electron as well as electron and ion can be ignored. In this paper, hydrodynamic equations without the energy equation are combined with Poisson's equation to propose the early picosecond plasma emission. It is predicted that there is an ultrashort anomalous emission of isothermal plasmas in the longitudinal direction due to the strongly anisotropic pressure and strong charge separation. The phenomenal has not been observed and reported.

In Sec. II, a self-similar two-dimension solution for a non-quasi-neutral plasma expansion with anisotropic pressure is obtained. In the solution, there are two important parameters: the ratio of the longitudinal temperature $T_\perp$ and the transverse temperature $T_\parallel$ of the plasma $\kappa^2 = T_\perp / T_\parallel$ and the electron-ion mass ratio $\mu = Z m_e / m_i$, where Z is the charge number of the ion, $m_e(m_i)$ is the electron (ion) mass. With the solution, it is found that the relationship between κ^2 and μ decides the shape of the ion front: a complicated surface or an ellipse.

For $\kappa^2 \in [0, \mu)$, the ion front is composed by a part of a hyperbolic and a small pointed projection at the center. In the critical case $\kappa^2 = \mu$, the ion front is a plane and a small pointed projection at the center. For $\kappa^2 \in (\mu, 1]$, the ion front is a part of an ellipse and the major axis is in the lower-temperature axis. The ion-velocity distribution at the ion front has also been given. However, for $\kappa^2 \in (0, 1)$, the major axis of the ion-velocity ellipse is in the higher-temperature axis. The difference of the angular-energy distribution from the known one [7,8] is that the energy is a delta function at the maximum angle of near 90° for $\kappa^2 \leq \mu$. The angle of the anomalous emission belongs to $[\arctan \mu^{-1/2}, \pi/2]$ and the duration of it is on the order of tens of femtoseconds due to the electron-electron and electron-ion collisions [9]. This anomalous phenomenal cannot been predicted with the three-dimension theory [3,4] for the quasineutral plasma expansion since the strong charge separation was ignored. For the scale time of nanoseconds [9], it is satisfied that $\kappa^2 > \mu$. Particularly, it is interesting for astrophysicist that anomalous positron emissions exist in the electron-positron plasma expansion with an anisotropic electron pressure.

II. ANISOTROPIC NON-QUASI-NEUTRAL PLASMA EXPANSION WITH STRONG CHARGE SEPARATION

A. Basic assumptions and equations

When an ultrashort laser pulse interacts with a solid target, electrons are generated in few femtoseconds with aniso-

*huangyongs@gmail.com

tropic temperatures: $T_{\parallel}$ in the x direction and $T_{\perp}$ in the y direction, which is perpendicular to x. For the laser intensity larger than 10^{16} W/cm^2, the temperature of laser-produced electrons is larger than several keV. Therefore, for the density of 10^{20}/cm^3, the electron-electron collision frequency and electron-ion collisions frequency are about 10^{12}/s. The higher the electron temperature, the smaller the collision frequency. We cut the time into pieces, each δt long. Therefore, in the short-time interval of δt, if δt^{-1} is larger than the electron-electron and electron-ion collision frequency, the plasma expansion can be assumed to be isothermal with characteristic temperatures: $T_{\parallel}$ and $T_{\perp}$. On the other hand, the temperatures of electrons can sustain constant due to the energy supply of laser pulse in the laser-pulse duration t_l.

Due to 1 eV=11 600 K, the electron temperature for laser intensity larger than 10^{17} W/cm^2 is higher than 10^8 K. With Debye-H*ü*chel's state equation [10], the electron-gas pressure

$$p = nk_BT_e - \frac{k_BT_e}{24\pi\lambda_{De}^3}, \tag{1}$$

in Gaussian units. The second part comes from the electrostatic term of free energy. For high temperature on the order of keV and the electron density of 10^{20}/cm^3, the ratio of the second part and the first part of the above equation is smaller than 10^{-7}. Therefore, the high-temperature electron-gas pressure is $p = nk_BT_e$.

In the time interval of about hundreds of femtoseconds after the solid target shot by an ultrashort laser pulse, the ions have not been accelerated efficiently and the ion temperature k_BT_i is about several eV, which is still high enough to make the viscosity term ($\propto T_i^{-3/2}$) negligible compared with the charge-separation field. However, $k_BT_i \ll k_BT_{\parallel}$ and $k_BT_i \ll k_BT_{\perp}$. Therefore, the ion pressure can also be neglected compared with the electron pressure in the two-fluid system. It is called the cold-ion assumption.

Based on the above analysis, in the ultrashort time interval of δt, we can assume that the pressure tensor of a two-dimension anisotropic plasma satisfies

$$\bar{P} = \begin{pmatrix} n_ek_BT_{\parallel} & 0 \\ 0 & n_ek_BT_{\perp} \end{pmatrix}. \tag{2}$$

In Eq. (2), since the electron temperature is high enough, the shear components are ignored for the convenience of the following analytic calculation, which is equivalent to neglecting the viscosity term in the two-fluid equations.

For convenience, the physical parameters: the time t, the length coordinate $x(y)$, the ion (electron) velocity $v_i(v_e)$, the electron field E, and the ion (electron) density $n_i(n_e)$ are normalized by the inverse of the equivalent plasma frequency $\omega_{pi0} = \sqrt{n_{e0}e^2/m_i\epsilon_0}$, the equivalent plasma Debye length $\lambda_{D0} = c_s/\omega_{pi0}$, the equivalent ion acoustic speed $c_s = \sqrt{Zk_BT_e/m_i}$, $E_0 = k_BT_e/e\lambda_{D0}$, and the reference hot-electron density n_{e0}, respectively, where m_i is the ion mass, Z is the charge number of the ion, e is the elemental charge, and $T_e = T_{\parallel} + T_{\perp}$ is an equivalent temperature.

Then the electric potential is normalized as

$$\phi = \frac{e\psi}{k_BT_e}, \tag{3}$$

where ψ is the physical potential. $t, x(y), v_i(v_e), E, n_i(n_e)$ are still used to represent the normalized parameters in the following discussion.

With the cold-ion assumption, the ion pressure gradient and ion viscosity are neglected. With Eq. (2), the electron viscosity can be ignored. Therefore, the two-fluid system without the energy equation is then normalized as

$$\frac{\partial n}{\partial t} + \frac{\partial(nv_x)}{\partial x} + \frac{\partial(nv_y)}{\partial y} = 0, \tag{4}$$

$$\frac{\partial v_{x_i}}{\partial t} + v_{x_i}\frac{\partial v_{x_i}}{\partial x_i} = -\frac{\partial \phi}{\partial x_i}, \tag{5}$$

$$\frac{\partial n_e}{\partial t} + \frac{\partial(n_ev_{e,x})}{\partial x} + \frac{\partial(n_ev_{e,y})}{\partial y} = 0, \tag{6}$$

$$\mu\left(\frac{\partial v_{e,x_i}}{\partial t} + v_{e,x_i}\frac{\partial v_{e,x_i}}{\partial x_i}\right) = \frac{\partial \phi}{\partial x_i} - \frac{\alpha_i}{n_e}\frac{\partial n_e}{\partial x_i}, \tag{7}$$

where $i = 1, 2, x_1 = x, \ x_2 = y$,

$$\alpha_1 = \frac{T_{\parallel}}{T_e}, \quad \alpha_2 = \frac{T_{\perp}}{T_e}, \tag{8}$$

and Poisson's equation is normalized as

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = n_e - Zn, \tag{9}$$

B. Analytical expressions of the ion front shape

With reference to the similarity transformation used by Huang *et al.* [7], the systems (4)–(9) can be transformed using

$$\tau = t, \quad \xi_1 = \frac{x}{R_1(t)}, \quad \xi_2 = \frac{y}{R_2(t)}, \tag{10}$$

$$v_{i,x}(v_{e,x}) = \frac{\partial x}{\partial \tau} = \xi_1 R_1', \quad v_{i,y}(v_{e,y}) = \frac{\partial y}{\partial \tau} = \xi_2 R_2', \tag{11}$$

$$n_{i(e)} = \frac{N_{i(e),1}(\xi_1,\xi_2)}{R_1^2} + \frac{N_{i(e),2}(\xi_1,\xi_2)}{R_2^2}, \tag{12}$$

where R_1, R_2, N_1, and N_2 need to be determined by solving the transformed hydrodynamic equations. The transformed continuity equation is

$$\left(\frac{R_1'}{R_1} - \frac{R_2'}{R_2}\right)\left(\frac{N_{i(e),1}}{R_1^2} - \frac{N_{i(e),2}}{R_2^2}\right) = 0, \tag{13}$$

which requires

$$\frac{R_2}{R_1} = \kappa, \tag{14}$$

where κ is a constant. Then the ion (electron) density is simplified to

$$n_{i(e)} = \frac{N_{i(e)}(\xi_1,\xi_2)}{R^2(t)}, \tag{15}$$

where

$$N_{i(e)} = N_{i(e),1} + \frac{N_{i(e),2}}{\kappa^2}, \tag{16}$$

$$R = R_1. \tag{17}$$

Solving the transformed motion equation of ions,

$$R_i'' R_i = -\frac{1}{\xi_i}\frac{\partial \phi}{\partial \xi_i}, \quad i = 1,2, \tag{18}$$

gives that

$$\int_0^{\alpha} \exp(\alpha_1^2) d\alpha_1 = \frac{\sqrt{\phi^0} t}{R_0}, \tag{19}$$

where

$$\alpha = \sqrt{\ln\left(\frac{R}{R_0}\right)}, \tag{20}$$

and

$$R_0 = R(t=0). \tag{21}$$

Therefore, the potential in the ion region satisfies

$$\phi = -\phi^0(\xi_1^2 + \kappa^2 \xi_2^2), \tag{22}$$

where ϕ^0 is a constant.

The transformed electron motion equation is

$$\mu R_i'' R_i \xi_i + \alpha_1 \frac{\partial \ln(N_e)}{\partial \xi_i} - \frac{\partial \phi}{\partial \xi_i} = 0, \quad i = 1,2. \tag{23}$$

Assuming

$$\kappa^2 = \frac{T_\perp}{T_\parallel}, \tag{24}$$

with Eq. (23), the electron density in the ion region satisfies

$$N_e(\xi_1,\xi_2) = N_{e,0} \exp\left[-(1+\mu)\phi^0 \frac{(\xi_1^2+\xi_2^2)}{\alpha_1}\right]. \tag{25}$$

Then from Poisson's equation, the ion density is $N_i = N_e + 4$, which shows the non-quasi-neutrality of the plasma. Equation (23) can be solved in the ion region due to the special form of the electric potential $\phi = \phi_1(\xi_1) + \phi_2(\xi_2)$.

However, beyond the ion front, the potential and electron density are governed by the motion equation of electrons and Poisson's equation together since the potential cannot be separated to $\phi_1(\xi_1) + \phi_2(\xi_2)$ and then the electron density cannot be solved from the motion equation solely.

By combining Eq. (23) and Poisson's equation, a two-order partial differential equation of the electron density can be achieved,

$$\alpha_1\left(\frac{\partial^2 \ln N_e}{\partial \xi_1^2} + \kappa^2 \frac{\partial^2 \ln N_e}{\partial \xi_2^2}\right) + 2(1+\kappa^2)\mu\phi^0 = N_e. \tag{26}$$

The first integral of it gives

$$\left(\frac{\partial Y}{\partial \xi_1}\right)^2 + \kappa^2\left(\frac{\partial Y}{\partial \xi_2}\right)^2 = \frac{2\exp(Y) - 4(1+\kappa^2)\mu\phi^0 Y}{\alpha_1} + C_0, \tag{27}$$

where C_0 is the first integral constant and $Y = \ln(N_e)$.

The physical condition requires that Y is a C^1 function and, therefore, the curve equation of the ion front is

$$\frac{\xi_1^2}{A^2} + \frac{\xi_2^2}{B^2} = D \exp\left[-\frac{(\xi_1^2+\xi_2^2)}{2D\phi^0}\right] + C_0, \tag{28}$$

$$A^{-2} = 1 - \mu\kappa^2, \quad B^{-2} = \kappa^2 - \mu, \quad D = \frac{\alpha_1}{2(1+\mu)\phi^{0,2}}, \tag{29}$$

where C_0 is a constant and $\phi^{0,2}$ is the square of ϕ^0. With Eq. (28), the curve of the ion front may be some part of a hyperbola, an ellipse, or a circle approximately for different ratio κ^2. A critical value of κ^2 is the mass ratio of electron and ion μ or the inverse of it.

Due to the assumed symmetrical form of the pressure, we only need to consider the cases for $\kappa^2 \in [0,1]$. Figure 1 shows the eight cases of the ion front for $\kappa^2 \in [0,1]$, which are obtained by solving Eq. (28) with $C_0 = 0$ and $\phi^0 = 1$. For $\kappa^2 > \mu$, the curve of the ion front is a part of an ellipse whose eccentricity is

$$e = \frac{(1-\kappa^2)(1+\mu)}{1-\kappa^2\mu}, \tag{30}$$

as shown by Fig. 1(a).

As $\kappa^2 \to \mu$, $e \to 1$. Specially, for $\kappa^2 = 1$, i.e., $T_\perp = T_\parallel$, the ion front is a part of a circle. This result consists with that given in [7,8] and can be used to predict the ion maximum energy and the angular ion distribution in the laser-thin interactions.

For $\kappa^2 \leq \mu$, the ion front is complicated and contains two parts: a part of a hyperbola and another curve, as shown by Fig. 1(b). For the part of the hyperbola, $\xi_2 \geq 2.5$ and the slope of the asymptote is $\sqrt{(1-\mu\kappa^2)/(\mu-\kappa^2)}$.

As $\kappa^2 \to \mu$, the slope trends to infinite and the ion front becomes a flat surface, as shown by the dash dot line in Fig. 1(b). This is the longitudinal anomalous plasma emission. For $\kappa = 0$, the emission angle is smallest and equal to $\arctan(\mu^{-1/2})$. Therefore, the range of the emission angle is $[\arctan(\mu^{-1/2}), \pi/2]$.

C. Velocity distribution at the ion front

Figure 2 shows the ion-velocity distribution at the ion front obtained from our two-dimension theory for different electron-temperature ratio $\kappa^2 \in [0,1]$. For different times, the shapes of the ion-velocity distribution are the same although the amplitudes are different.

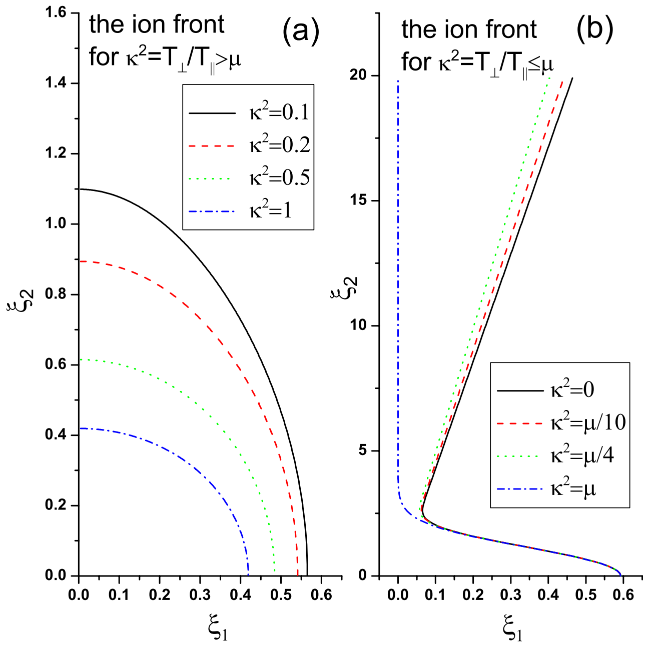

FIG. 1. (Color online) (ξ_1,ξ_2) at the ion front for different κ^2 for $\phi^0=1$ and $C_0=0$ in Eq. (28). (a) and (b) are obtained from the solutions of Eq. (28). In (a), $\kappa^2>\mu$, where $\mu=Zm_e/m_i$, Z is the ion charge number, and $m_e(m_i)$ represents the mass of electron(ion). In these cases, the curve of the ion front is a part of an ellipse, especially, a circle for $\kappa^2=1$. The major axis of the ellipse is in the lower-temperature direction. In (b), $\kappa^2\leq\mu$. In these cases, the ion front contains two parts. For $\xi_2\geq 2.5$, the ion front is a part of a hyperbola or a plane ($\kappa^2=\mu$). For $\xi_2\leq 2.5$, the ion front is a small pointed projection.

For $\kappa^2\in(0,\mu)$, the velocity distribution has two parts: ellipselike at small v_2 and hyperboliclike at large v_2, which is shown by the dot line or the dash line in Fig. 2(a). The solid line in Fig. 2(a) shows all the ions move toward the x direction since the temperature in the y direction is zero. The dash dot line in Fig. 2(a) shows the critical case: $\kappa^2=\mu$, in which the velocity in the x direction at large v_2 is zero and all the ions move in the y direction.

For $\kappa^2\in(\mu,1]$, the ion-velocity distribution is a part of an ellipse, whose eccentricity is $\sqrt{\mu(\kappa^{-2}-\kappa^2)/(1-\mu\kappa^{-2})}$ and the major axis is in the high-temperature direction x. At $\kappa^2=1$, the ion-velocity distribution is a part of a circle and isotropic, as shown by the dash dot line in Fig. 2(b).

D. Angular-energy distribution at the ion front

Figure 3 shows the angular-energy distribution for $\kappa^2\in[0,1]$. With it, the facts are indicated:

(1) For $\kappa^2\in[0,\mu]$, the energy decreases with angle except for the maximum angle of near 90°. At the maximum angle, the energy is a delta function with the peak value of about $0.62E_0$. Especially, the maximum angle is 90° for the critical point $\kappa^2=\mu$. This anomalous plasma emission happens since the large Coulomb space-charge field dominates in the longitudinal direction and is counteracted partly by the gradient of the pressure in the transverse direction. As seen in Fig. 3, the anomalous emission angle belongs to $[\arctan(\mu^{-1/2}),90°]$, which is obtained above from Fig. 1.

(2) For $\kappa^2\in(\mu,1)$, the ion energy decreases with angle monotonically. At $\kappa^2=1$, the energy is a constant function with respect to angle.

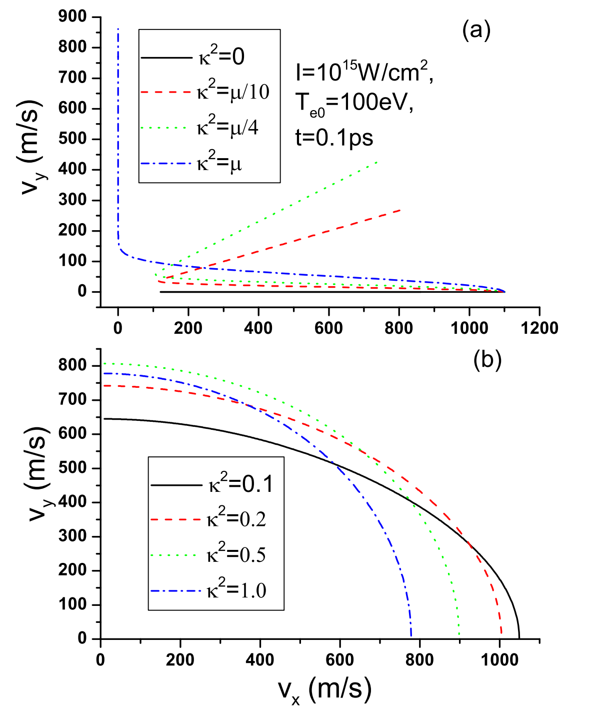

FIG. 2. (Color online) The ion-velocity distribution at the ion front for $I=10^{15}$ W/cm², $\lambda=0.8$ nm, and $\kappa^2\in[0,1]$; $\phi^0=1$ in Eq. (28), $\alpha(t)=0.003$ at $t=0.1$ ps. (a) The ion-velocity distribution at the ion front for $\kappa^2\leq\mu$. (b) The ion-velocity distribution at the ion front for $\kappa^2>\mu$.

E. Duration of the anomalous emission

As it is well known, when an ultraintense laser pulse interacts with solid materials, electrons with anisotropic tem-

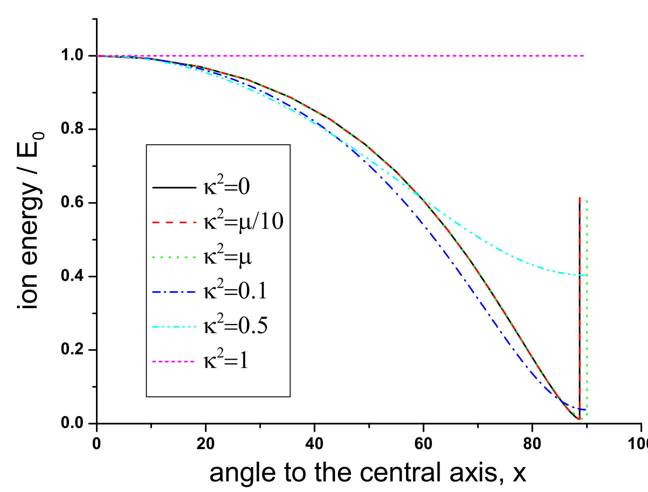

FIG. 3. (Color online) The ion energy distribution at the ion front VS the angle to the central axis x for $\kappa^2\in[0,1]$, $\phi^0=1$ in Eq. (28). In this figure, the energy for any angle is normalized by E_0 for each κ^2, where E_0 is the ion energy at the central axis.

peratures are generated in a time interval on the order of few femtoseconds. As pointed out by Eq. (2) and Fig. 1 shown by Ferrante *et al.* [9], the transverse temperature of electrons $T_{\parallel}$ would decrease and the longitudinal temperature $T_{\perp}$ would increase due to the electron-electron and electron-ion collisions. They gave the growth rate of the temperatures [Eqs. (3) and (4) in Ref. [9]] by solving the kinetic equation [Eq. (2) in Ref. [9]] under the bi-Maxwellian distribution assumption. In fact, if the initial state satisfies $T_{\parallel} < \mu T_{\perp}$, after a time interval on the order of tens of femtoseconds, $T_{\parallel} > \mu T_{\perp}$. After a time on the order of few or tens of picoseconds, the two temperatures tend to be the same.

In order to show the fact, we assume that the initial state is $T_{\perp,0}=0$ and $T_{\parallel,0}=100$ eV. Set the electron density to be $10^{19}/\text{cm}^3$ (which is about the value measured by relevant experiments [8]), with Eqs. (3) and (4) in Ref. [9], $T_{\perp}$ increases from 0 eV to $\mu T_{\parallel,0}$, i.e., $\kappa^2=\mu$, at about 44 fs. After several nanoseconds, it is satisfied that $\mu T_{\parallel} \ll T_{\perp} \leq T_{\parallel}$, i.e., $\mu \ll \kappa^2 \leq 1$.

At the very beginning time, the ion front is a flat surface with a small pointed projection at the center shown by the dash dot line in Fig. 1(b). There is the longitudinal anomalous plasma emission in the interval on the order of tens of femtoseconds. After that, $\kappa^2 > \mu$, and then the ion front develops to a part of ellipses shown by the solid, dash, and dot lines step by step in Fig. 1(a). At last, the ion front trends to a part of a circle shown by the dash dot line in Fig. 1(a).

F. Anomalous positron emission in the pair-plasma expansion

An interesting deduction is as follows: the longitudinal positron emission happens in the electron-positron plasma expansions for $\kappa < 1$ since $\mu=1$, if the positron temperature is far smaller than the electron temperature. Therefore, the anomalous positron emissions exists if the electron pressure is anisotropic, which is easy to be satisfied.

However, it is required that the positron temperature is small enough compared with electron temperatures in the initial stage of the expansion. In this case, the duration of the emission is about the inverse of the electron-positron or electron-electron collision frequency.

The anomalous positron emission may exist in the space and observed in the space detection since the space plasma is rarefied and then the duration of the emission will be long enough to be observed.

III. CONCLUSION

In conclusion, the analytic expression for the ion front of plasma expansion with anisotropic pressure and strong charge-separation field is predicted. The ion-velocity distribution at the ion front and angular-energy distribution at the ion front are both given. It needs to be concerned that the energy is a delta function at the maximum angle of near 90° and the plasma emits anomalously due to the longitudinal Coulomb explosion for $\kappa^2 \in [0,\mu]$, as shown by the solid, dash, and dot lines in Fig. 3. This anomalous phenomenon has not been observed since the strongly anisotropic plasma is difficult to be generated or even if it is generated in the laser-plasma interaction, the duration of the emission is too short to be observed with the existing technologies. If we can generate a strongly anisotropic plasma that can sustain a period long enough or we can master an ultrafast detection technology, the anomalous emission will be observed and then yields significant applications.

However, anomalous positron emissions exist in the electron-positron plasma expansion with an anisotropic electron pressure if the positron temperature is small enough in the early stage of the expansion. The emission duration would be longer than several picoseconds.

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